A PRIORITY BASED FUZZY GOAL PROGRAMMING APPROACH FOR
MULTI-OBJECTIVE SOLID TRANSPORTATION PROBLEM

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Abstract: This paper presents priority based fuzzy goal programming approach to solve multi-objective solid transportation problem. The proposed approach attempts to minimize the transportation cost, environmental cost and congestion time cost. Linear membership function is constructed by determining the individual best solution of the objective functions subject to the system of constraints. Then fuzzy goal programming approach is used for achieving highest degree of each of the objective goals by minimizing negative deviational variables. Then sensitivity analysis with the variations of the priority structure is performed to identify the most appropriate priority structure in the decision making context by using distance function. A numerical example is solved to show the efficiency of the proposed approach.

Keywords: Goal programming, multi-objective solid transportation problem, priority based fuzzy goal programming, transportation cost, environmental cost, congestion time cost.

1. INTRODUCTION

In today’s highly competitive market the pressure on organizations to find the better ways to create and deliver values to customers is increasing more and more. How and when to send the products to the customers in the quantities they want in a cost effective manner has become more challenging. Transportation model provides powerful framework to meet this challenge.

Hitchcock [12] first introduced the basic transportation problem in 1941. The classical transportation problem (Hitchcock transportation problem) is considered to be one of the subclasses of linear programming problems, in which all the constraints are of the equality type.

The solid transportation problem (STP) is a generalization of the well-known transportation problem (TP) in which three-dimensional properties are taken into account in the objective and constraint set instead of source and destination. The STP was first stated by Shell [8]. In many industrial problems, a homogeneous product is delivered from an origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, etc. These conveyances are taken as the third dimension. A solid transportation problem can be converted to a classical transportation problem by considering only a single type of conveyance. Haley [21] introduced the solution procedure of STP which is an extension of the modified distribution method. Patel and Tripathy [16] developed a computationally superior method for a STP with mixed constraints.

Because of the complexity of the social and economic environment, the real life problems are modelled with multi-objective functions which are measured in different respects and they are non-commensurable and conflicting in nature. Furthermore, it is frequently difficult for the decision maker to combine the objective functions in one overall utility function. Charnes and Cooper [6] first discussed on various approaches to solutions of managerial level problems involving multiple conflicting objectives. The multi objective transportation problem (MOTP) in a crisp environment was extensively studied in refs. [19, 38].

It is well known that the priority based goal programming (GP) is one of the powerful techniques in field of multi criteria decision-making problems with multiple and conflicting objectives. Ijiri [17] introduced priority based GP at first in 1965. Ignizio [18], Lee [23], Steuer [43] and other researchers developed priority based GP and they successfully applied GP to various real-life decision-making problems. In the proposed priority based GP, the ranking of goals are grouped according their priorities for achieving
the respective aspiration levels in the decision-making context. The goals, which are considered equal importance, belong to the same priority level. The goals at the first priority level are considered to be infinitely more important than the goals at the second priority level. The goals at the second priority level are considered to be infinitely more important than the goals at the third priority level and so on. Since the goals are generally conflicting in nature, differential weights are assigned to their relative importance for achieving the respective desired values.

In a solid transportation problem more than one objective is normally considered. It is often difficult to estimate the actual penalties (e.g., transportation cost, delivery time, quantity of goods delivered, under-used capacity, etc.), demands, availabilities, the capacities of different modes of transport between origins and destinations. Depending upon different aspects, they fluctuate due to uncertainty in judgement, lack of evidence, insufficient information, etc. Sometimes it is not at all possible to get relevant precise data. So these parameters are assumed to be flexible/imprecise in nature i.e., uncertain in a non-stochastic sense and may be represented by fuzzy numbers.


The existing procedures in the literature (see Deb[7]; Rao [36]) for solving multi-objective transportation problems can be divided into two categories.
those are generating all the sets of efficient solutions (see Ringuest and Rinks, [38]; Gen et al., [14]) and the second category represents the procedure of using an additional criterion to obtain the best compromise solution among the set of efficient solutions (see Rakesh Varma & Biswas, [39]; Bit et al., [2]; and Sy-Ming Gun & Yan - Kuen Wu, [45]) developed various functions to achieve direct compromise solution without developing and testing all the Pareto solutions.

Transport improvements promote economic growth and social development by increasing mobility and improving access to resources and markets. However, in recent years, there has been growing recognition of the need to promote sustainability, sustainable development, and sustainable transport in order to bring about improvements in transport systems and policies.

The environmental impact of transport is significant because it is a major user of energy, and burns most of the world's petroleum. This creates air pollution, including nitrous oxides and particulates, and is a significant contributor to global warming through emission of carbon dioxide, for which transport is the fastest-growing emission sector. By subsector, road transport is the largest contributor to global warming.

Economic measures are regarded as a promising way for reducing the environmental impacts of transport. Such measures include not only direct operating (internal) costs but also costs resulting from environmental impacts, congestion, accidents etc. (the so-called external costs). These external costs are not covered by travelers but are imposed on the whole society, other regions or future generations. As a result, current transport prices do not reflect the full costs caused by transport. However, these prices influence mobility behavior by setting the financial scope in which people realize their travel. If they do not include the external cost components people will not include them into their travel decisions either. Thus, pricing measures for internalizing the external costs, i.e. making the users pay for all the costs they cause, are a powerful instrument to change these decisions and reduce the environmental impacts of transport. The estimation of external costs is uncertain, i.e. consist of a range rather than a single value.

Traffic congestion is a widely recognized transport cost. It is a significant factor in transport system performance evaluation and affects transport planning decisions. Traffic congestion is common in large cities and on major highways and it imposes a significant burden in lost time, uncertainty, and aggravation for passenger and freight transportation. The European UNITE project estimated the costs of traffic congestion in the UK to be £15 billion/year ($23.7 billion/year) or 1.5% of GDP (Nash et al., [31]). For France and Germany the estimates were 1.3% and 0.9% of GDP respectively. The Texas Transportation Institute conducts an annual survey of traffic congestion in US urban areas. According to the 2009 report, in 2007. Congestion caused an estimated 4.2 billion hours of travel delay and 2.8 billion gallons of extra fuel consumption with a total cost of $87 billion (Schrank and Lomax, [42]). With a US GDP of $14.1 trillion in 2007 the cost amounted to 0.6% of GDP. The average cost per traveler in the urban areas studied was $757. Most of the costs of traffic congestion are borne by travelers collectively but, because individual travelers impose delays on others, they do not pay the full marginal social cost of their trips and therefore create a negative externality. Most economists have supported congestion pricing although many have been concerned about the details of implementation (Lindsey, [26]). Congestion pricing has a big advantage over other transportation demand management policies in that it encourages travelers to adjust all aspects of their behavior: number of trips, destination, mode of transport, time of day, route, and so on, as well as their long-run decisions on where to live, work and set up
business. For decades congestion pricing remained largely an ivory-tower idea, but interest gradually spread outside academia and congestion pricing has come into limited practice. In recent years, with strong economic development and rapid urbanization in China, traffic congestion has become a common phenomenon in urban areas. Focusing on the high-density development of land use in China, the urban population has become highly concentrated and the number of vehicles has also increased exponentially. Traffic congestion problems have become more and more serious in most of the major cities in China.

The consideration of environmental cost and congestion time cost are essentially changing the transportation policy in developing countries. The multi-objective solid transportation problem with uncertain environmental cost and congestion time cost has not been studied extensively in the literature. So, by taking these costs into consideration along with the transportation cost for some of the major cities in China, this paper provides the solution procedure for a priority based fuzzy goal programming approach for multi-objective solid transportation problem.

For model formulation of the problem, the objective functions for the fuzzy goals are defined first. Subsequently, the membership functions are transformed into fuzzy membership goals by means of assigning the highest degree (unity) as the aspiration level, and introducing the negative and positive deviational variables to each of them. As positive deviation from any fuzzy goals implies the full achievement of the membership value, the authors have used only negative deviational variables in the achievement function and minimized the negative deviational variables to get a satisfying solution [33]. In the priority based FGP solution technique, first the goals at the highest priority level are taken into consideration for achievement of their aspiration levels according to their relative importance of weights at that priority level. Next, the achievement of the goals of the very next higher level is taken into consideration and the process goes on until the last priority level is considered.

In the solution process, sensitivity analysis using the variations of the priority structure of the goals is performed to show how the optimal solution is sensitive to the changes in the priority structure. To identify the appropriate priority structure under which the most satisfying decision is reached in the decision-making environment, the Euclidean distance function is applied.

This paper is organized as follows In section 2 preliminary concepts of fuzzy set theory are given. Fuzzy multi-objective solid transportation problem is defined in section 3. In section 4, the solution procedure for MOSFTP using priority based fuzzy goal programming is discussed. Section 5 provides a numerical example to illustrate the proposed method. In section 6, conclusions are drawn.

2. PRELIMINARIES

Though there are a number of ways of defining fuzzy numbers, for the purpose of this paper we adopt the following definition. Throughout this paper \( \mathbb{R} \) stands for the set of all real numbers. \( E \) stands the set of fuzzy numbers, \( u \) expresses a fuzzy number and \( \mu(x) \) for its membership function, \( \forall x \in \mathbb{R} \).

**Definition 2.1 :** [30]
A fuzzy number is a fuzzy set like \( \mu : \mathbb{R} \to [0, 1] \) which satisfies :
1. \( \mu \) is upper semi-continuous.
2. \( \mu(x) = 0 \) outside some interval \([a, d]\).
3. There are real numbers \( a, b \) such that \( a \leq b \leq c \leq d \) and
\((i)\) \(\mu(x)\) is monotonically increasing on \([a, b]\).

\((ii)\) \(\mu(x)\) is monotonically decreasing on \([c, d]\).

\((iii)\) \(\mu(x) = 1, b \leq x \leq c\).

The membership function \(u\) can be expressed as

\[
\mu(x) = \begin{cases} 
\mu_L(x), & a \leq x \leq b \\
1, & b \leq x \leq c \\
\mu_R(x), & c \leq x \leq d \\
0, & \text{otherwise}
\end{cases}
\]

where \(\mu_L : [a, b] \rightarrow [0, 1]\) and \(\mu_R : [c, d] \rightarrow [0, 1]\) are left and right membership functions of fuzzy number \(\mu\). An equivalent parametric form is also given in [37] as follows.

**Definition 2.2 :** [37] A fuzzy number be in parametric form is a pair \((u, \bar{u})\) of functions \(u(r), \bar{u}(r), 0 \leq r \leq 1\), which satisfy the following requirements.

1. \(u(r)\) is bounded monotonic increasing left continuous function.
2. \(\bar{u}(r)\) is bounded monotonic decreasing left continuous function.
3. \(u(r) \leq \bar{u}(r), 0 \leq r \leq 1\).

The trapezoidal fuzzy number \(u = (x_0, y_0, \sigma, \beta)\) with two defuzzifier \(x_0, y_0\) and left fuzziness \(\sigma > 0\) and right fuzziness \(\beta > 0\) is a fuzzy set where the membership function as

\[
\mu(x) = \begin{cases} 
\frac{1}{\sigma}(x-x_0+\sigma), & x_0 - \sigma \leq x \leq x_0 \\
1, & x \in [x_0, y_0] \\
\frac{1}{\beta}(y_0-x+\beta), & y_0 \leq x \leq y_0 + \beta \\
0, & \text{otherwise}
\end{cases}
\]

and its parametric form is \(u(r) = x_0 - \sigma + \sigma r, \bar{u}(r) = y_0 + \beta - \beta r\) provided that \(x_0 = y_0\), then u is a triangular fuzzy number and we write \(u = (x_0, \sigma, \beta)\).

**2.3. Ranking of Trapezoidal Fuzzy Numbers [41]**

For an arbitrary trapezoidal fuzzy number \(u = (x_0, y_0, \sigma, \beta)\) with parametric form \(u = (u(r), \bar{u}(r))\). The magnitude of the trapezoidal fuzzy number is \(u\) is defined as

\[
\text{Mag}(u) = \frac{1}{2} \int \left[ (u(r)+\bar{u}(r)+x_0+y_0)f(r) \right] dr \quad \ldots (2.3.1)
\]

where the function \(f(r)\) is a non-negative and increasing function on \([0, 1]\) with \(f(0) = 0\) \(f(1) = 1\) and \(\int_0^1 f(r) dr = \frac{1}{2}\). Obviously function \(f(r)\) can be considered as a weighting function. In actual applications, function \(f(r)\) can be chosen according to the actual situation. In this paper we use \(f(r) = r\). Obviously, the magnitude of a trapezoidal fuzzy number \(u\) which is defined by (2.3.1), synthetically reflects the information on every membership degree. The resulting scalar value is used to rank the fuzzy numbers. In other words \(\text{Mag}(u)\) is used to rank fuzzy numbers. The larger \(\text{Mag}(u)\), the larger fuzzy number.
Therefore for any two trapezoidal fuzzy numbers $u$ and $v \in E$, we define the ranking of $u$ and $v$ by the $\text{Mag}(.)$ on $E$ as follows:

(1) $\text{Mag}(u) > \text{Mag}(v)$ if and only if $u \prec v$.

(2) $\text{Mag}(u) < \text{Mag}(v)$ if and only if $u \succ v$.

(3) $\text{Mag}(u) = \text{Mag}(v)$ if and only if $u \sim v$.

Then we formulate the order $\geq$ and $\leq$ as $u \geq v$ if and only if $u > v$ or $u \sim v$, $u \leq v$ if and only if $u < v$ or $u \sim v$.

3. FUZZY MULTI-OBJECTIVE SOLID TRANSPORTATION PROBLEM

Consider $m$ sources and $n$ destinations in a solid transportation problem at each source let $S_i$ be the amount of a homogeneous product we want to transport to $n$ destinations to satisfy the demand for $d_j$ units of the product. Let $e_l$ denote the units of this product that can be carried by $p$ different modes of transportation called conveyance, such as car, truck, train, airfreight and ocean shipping. A penalty value $C_{ijl}$ may represent unit shipping cost, environmental cost, congestion time cost or others of a product from source $i$ to destination $j$ by means of the $l$th conveyance. We need to determine a feasible way of shipping the available amounts to satisfy the demand such that the total cost is minimized.

Let $x_{ijl}$ denote the number of units to be transported from source $i$ to destination $j$ through conveyance $l$. The multi-objective solid transportation problem is of the following mathematical form:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{p} C_{ijl} x_{ijl}$$

subject to

$$\sum_{j=1}^{n} x_{ijl} \geq s_i, \ i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ijl} \geq d_j, \ j = 1, 2, \ldots, n \quad \ldots (3.1)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijl} \leq e_l, \ l = 1, 2, \ldots, p$$

$$x_{ijl} \geq 0, \ \forall \ i, j, l.$$  

Intuitively, if any of the parameters $C_{ijl}$, $s_i$, $d_j$ or $e_l$ or all the parameters are fuzzy, the total cost becomes fuzzy as well suppose the unit shipping cost, environmental cost, congestion time cost $C_{ijl}$, supply $S_i$, demand $d_j$ and conveyance capacity $e_l$ are represented by trapezoidal fuzzy numbers.

Then the fuzzy objection function

$$\text{Min } \tilde{F}(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{p} \tilde{C}_{ijl} x_{ijl}$$

which is to minimized together with the following constraints, constitute the multi-objective fuzzy solid transportation problem.

subject to

$$\sum_{j=1}^{n} x_{ijl} \geq \tilde{s}_i, \ i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijl} \geq \tilde{d}_j, \ j = 1, 2, \ldots, n$$
3.1. ENVIRONMENT COST AND CONGESTION TIME COST IN TRANSPORTATION

The consideration of environmental cost is essentially changing the transportation policy in developing countries and developed countries. The European conference on Ministers of Transport (1998) urged European Governments to develop new instruments to incorporate externalities and environmental costs in transport management accounting. Proceedings of Conference on transportation in developing countries (1998) said that, Transportation services produced unwanted by-products (Air-pollution, global warming, accidents and congestion). The cost of these negative externalities is very high around 6-7 percent of GDP and they need to be addressed. Therefore, environmental concerns have highlighted the importance of sustainable transport design. The estimation of external costs is uncertain, i.e. consist of a range rather than a single value. Moreover these costs are not covered by travelers but are imposed on the whole society, other regions or future generations. As a result, current transport prices do not reflect the full costs caused by transport. By making the users to pay for all the costs they cause, are a powerful instrument to change their travel decisions and reduce the environmental impacts of transport. Furthermore, the quantification of environmental externalities involves many uncertainties and heavily depends on the temporal and regional conditions. Thus we will make use of Todd Litman, Climate Change Emission Valuation for Transportation Economic Analysis, 22 February 2012, Victoria Transport Policy Institute (www.vtpi.org) to evaluate the environmental costs and congestion time costs associated with transportation.

4. SOLUTION PROCEDURE

Using the ranking procedure described in section 2, the problem defined in section 3 can be reformulated as

\[ \min F^k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{l=1}^{p} M(C^k_{ijl})x_{ijl} \]

which is to be minimized together with the following constraints, constitute the multi-objective fuzzy solid transportation problem.

s.t \[ \sum_{j=1}^{n} \sum_{l=1}^{p} x_{ijl} \leq M(\hat{s}_i), i = 1, 2, \ldots, m \]

\[ \sum_{i=1}^{m} \sum_{l=1}^{p} x_{ijl} \geq M(\hat{d}_j), j = 1, 2, \ldots, n \] \ldots (4.1)

\[ \sum_{i=1}^{m} \sum_{l=1}^{p} x_{ijl} \leq M(\hat{c}_j), l = 1, 2, \ldots, p \]

\[ x_{ijl} \geq 0, \ \forall \ i, j, l. \]

To formulate the fuzzy programming model of MOSFTP, the objective functions would be transformed into fuzzy goals by means of assigning an imprecise aspiration level to each of the objectives. The optimal solution of each objective function of the DMU when calculated in isolation would be considered as the aspiration level of the corresponding fuzzy goal.
To define the membership function of MOSFTP let \( U_k, l_k \) be the lower and upper bounds of the objective function \( F^k(x) \). These values are determined as follows: Calculate the individual minimum of each objective function as a single objective transportation problem subject to the given set of constraints. Let \( X_1, X_2, \ldots, X^K \) be the respective optimal solutions for the \( K \) different transportation problems and evaluate each objective function at all these \( K \) optimal solutions. It is assumed that at least two of these solutions are different for which the \( k \)-th objective function has different bounded values. Let \( U_K = \max F^K(x), K = 1, 2, \ldots, K \).

It is quite natural that objective values which are equal to or larger than \( U_K, K = 1, 2, \ldots, K \) should be absolutely acceptable to the DMU. Then the fuzzy goals appear in the form

\[
F^k(x) \geq U_K, \quad K = 1, 2, \ldots, K.
\]

Using the individual best solutions, we find the value of all the objective functions at each best solution and construct a payoff matrix as:

| \( F^1(x) \) | \( F^2(x) \) | \( \cdots \) | \( F^K(x) \) |
| \( F^1(x_1) \) | \( F^2(x_1) \) | \( \cdots \) | \( F^K(x_1) \) |
| \( F^1(x_2) \) | \( F^2(x_2) \) | \( \cdots \) | \( F^K(x_2) \) |
| \( \vdots \) | \( \vdots \) | \( \cdots \) | \( \vdots \) |
| \( F^1(x_K) \) | \( F^2(x_K) \) | \( \cdots \) | \( F^K(x_K) \) |

The maximum value of each column of \( F^K(x) \), \( K = 1, 2, \ldots, K \) provides the upper bound or aspired level of achievement for the objective goal. The minimum value of each column provides lower bound or lowest acceptable level of achievement for the objective goal.

The membership function of the \( K \)-th objective function can be defined as

\[
\mu_K[F^K(x)] = \begin{cases} 1, & \text{if } F^K(x) \geq U_K \\ \frac{U_K - F^K(x)}{U_K - L_K}, & \text{if } L_K < F^K(x) < U_K \\ 0, & \text{Otherwise} \end{cases}, \quad k = 1, 2, \ldots, K
\]

4.1 Priority based FGP Model of MOSFTP

The problem discussed in section (4) reduces to the following problem

\[
\text{Max } \mu_K[F^K(x)], \quad K = 1, 2, \ldots, K \quad \ldots (4.1.1)
\]

Subject to the constraints (4.1)

Now, the achievement of the highest degree (unity) of a membership function implies absolute achievement of the aspired level of the associated fuzzy goal. So membership goal corresponding to the \( K \)-th membership function with unity as the aspiration level can be presented as:

\[
\mu_K[F^K(x)] + d_{k-} - d_{k+} = 1 \quad \ldots (4.1.2)
\]

Here \( d_{k-} (\geq 0) \) and \( d_{k+} (\geq 0) \) represent the negative and positive deviational variables respectively. It may be noted that any over deviation from a fuzzy goal indicates the full achievement of the membership value. Then according to Pramanik and Roy[33], negative deviational variables are to be minimized in order to get satisfying solution. Therefore (4.1.2) can be reformulated as

\[
\mu_K[F^K(x)] + d_{k+} \geq 1 \quad \ldots (4.1.3)
\]
Therefore, under the framework of min-sum GP the priority based FGP model of the problem can be explicitly formulated as

Find \( X \) so as to

Minimize \( \bar{\phi} = \left[ P_1(d') , P_2(d') , \ldots , P_J(d') \right] \) \hspace{1cm} \ldots (4.1.4)

subject to

\[
\sum_{j=1}^{n} \sum_{l=1}^{p} x_{ijl} \leq M(\bar{S}_i) , \quad i = 1, 2, \ldots , m
\]

\[
\sum_{i=1}^{m} \sum_{l=1}^{p} x_{ijl} \geq M(\bar{D}_j) , \quad j = 1, 2, \ldots , n
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{p} x_{ijl} \leq M(\bar{E}_l) , \quad l = 1, 2, \ldots , p
\]

\[
x_{ijl} \geq 0 , \quad \forall i, j, l.
\]

\[
d_k \geq 0
\]

Here \( \bar{\phi} \) represents the vector of \( J \) priority achievement functions. \( P_j(d') \) is a linear function of the weighted negative deviational variables, where \( P_j(d') \) is of the form

\[
P_j(d') = \sum_{K=1}^{K} W_{jk} d_{jk} , \quad K = 1, 2, \ldots , K, \ j < K
\]

\[
\ldots (4.1.5)
\]

Here \( d_{jk} \) is renamed for \( d_k \) to represent \( j^{th} \) priority level. \( W_{jk} (0 \leq W_{jk} \leq 1) \) is the numerical weight corresponding to \( d_{jk} \). \( W_{jk} \) represents the weight of importance of achieving the aspired level of the \( K^{th} \) goal relative to other goals, which are grouped together at the \( j^{th} \) priority level. DMU may provide the numerical weight \( W_{jk} \) or normalized weight according to the needs, desires and practical situation of the decision making situation. Here, the \( j^{th} \) priority factor \( P_j \) is assigned to the set of commensurable goals that are grouped together in the problem formulation. In the preemptive priority FGP, \( j^{th} \) priority \( P_j \) is preferred to the next priority \( P_{j+1} \) regardless of any weight associated with \( P_{j+1} \).

The priority factors have the relationships

\[
P_1 \gg P_2 \gg P_3 \ldots \gg P_j \ldots \gg P_J
\]

\[
\ldots (4.1.6)
\]

Here ‘\( \gg \)’ denotes much greater than, ie the membership goals at the first priority level (\( P_1 \)) are achieved to the maximum possible before the set of membership goals at the second priority level (\( P_2 \)) is considered.

It is to be noted that if all the fuzzy goals are considered as equally important in a decision making context, the priority based FGP model (4.1.4) will be transformed into the weighted FGP model. It is to be noted that “too many” different priority structure can increase the computational burden to the DMU. If \( J \) be the total priority levels, then \( J \) priority structure may be involved there. However, in practice two to five priority levels are important to the DMU in the decision making situation and the conflict of assigning priorities arises at the most three priority levels. (Ignizio, J.P., 1976)

4.2. Selection of Appropriate Priority Structure: Use of Euclidean Distance Function

In the priority based FGP approach, the priorities are assigned to the goals on the basis of the importance of achieving their aspired levels in the decision context. However it is worthy to observe that in the highly conflicting decision situation, the decision making
unit feels confused with regard to assigning the appropriate priority structure for achieving their aspired goals, where the decision changes with the change of priorities to the goals. Consequently, a decision deadlock arises frequently in the decision making situations.

To deal with such problems, in the proposed MOSFTP formulation, the concept of Euclidean distance function for group decision analysis introduced by Yu, 1973, can be applied for measuring the ideal point dependent solution, to identify the appropriate structure of the goals under which most satisfying solution is achieved.

In the present FGP formulation of the MOSFTP, as the highest membership value of each fuzzy goal is unity, the ideal point would be a vector with each element equal to unity. Suppose S be the total number of different possible priority structures that arise in the decision situation, the Euclidean distance function can be represented as

\[ D^j = \left( \sum_{K=1}^{K} (1 - \mu^j(F^K(x)))^2 \right)^{1/2} \]

where \( \mu^j(F^K(x)) \) represents the achieved membership value of the \( K^{th} \) goal under the \( j^{th} \) priority structure of the goals. Now the solution for which the Euclidean distance function is minimal would be the most satisfying solution. Here it can be easily realized that the solution that is closest to the ideal point must correspond to \( \min_{j=1,2,\ldots,J} D^j = D^M \)

where \( 1 \leq M \leq J \). Then the \( M^{th} \) priority structure can be identified as an appropriate priority structure to achieve the most satisfying solution.

5. NUMERICAL EXAMPLE

Consider the Numerical Example given in [54], in which the problem of a concrete transportation problem at Xiluodu Hydropower Station is described. In order to build infrastructures in four destinations i.e. Leibo, Baisha, Daguar and Baiheaten the concrete is needed from Shifu, Yongshan and Puerdu. There are three kinds of conveyances: large trucks, medium trucks, and small trucks. There are five large trucks with 8 m\(^3\) transportation capacity for each one. There are eight medium trucks with 5 m\(^3\) transportation capacity for each one. There are five small trucks with 3 m\(^3\) transportation capacity for each one. For convenience, large truck, medium trucks and small truck are denoted by conveyance 1, conveyance 2 and conveyance 3. Thus, the total transportation capacities of conveyance 1, conveyance 2 and conveyance 3 are 40 m\(^3\), 40 m\(^3\) and 15 m\(^3\), respectively. In their work, transportation cost, transportation time and conveyance capacity are considered as deterministic parameters.

But in a real world situation, these parameters are uncertain due to some uncontrollable factors. Moreover, environment cost and congestion time cost play important role in a transportation decision making problems.

So in our numerical example all these costs are taken into consideration, and they are represented as trapezoidal fuzzy numbers which are given in the following table.

<table>
<thead>
<tr>
<th>Source</th>
<th>Supply capacity</th>
<th>Destination</th>
<th>Demand</th>
<th>Conveyance</th>
<th>Capacity</th>
</tr>
</thead>
</table>

Table 1

Estimated values of supply capacity of sources, demand of destinations and capacity conveyances (m\(^3\)/day).
| Shuifu       | 20 40 5 4 | Leibo    | 10 30 2 5 | 1 | 30 50 10 10 |
| Yongshan    | 20 40 5 4 | Baisha   | 10 30 2 5 | 2 | 30 50 10 10 |
| Puerdu      | 30 50 6 3 | Daguan   | 10 30 2 5 | 3 | 12 18 3 3  |
|             |           | Baihetan | 20 40 5 4 |  |              |

Unit transportation cost by conveyances (yuan/m³).

<table>
<thead>
<tr>
<th>Conveyance 1</th>
<th>Leibo</th>
<th>Baisha</th>
<th>Daguan</th>
<th>Baihetan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shuifu</td>
<td>[290 310 20 30]</td>
<td>[310 330 10 20]</td>
<td>[330 350 5 15]</td>
<td>[310 330 10 20]</td>
</tr>
<tr>
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<th>Daguan</th>
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**UNIT ENVIRONMENTAL COST** (Yaun / m³)

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<tr>
<td>Shuifu</td>
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<td>[1 3 1 2]</td>
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<tr>
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<td>[128 130 2 4]</td>
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</thead>
<tbody>
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<tr>
<td>Puerdu</td>
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<td>[219 221 4 5]</td>
<td>[193 195 2 6]</td>
<td>[214 216 3 5]</td>
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**UNIT CONGESTION TIME COST** (m³/yaun)

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<td>[37 39 2 4]</td>
<td>[6 8 2 5]</td>
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</table>
Subsequently following the proposed procedure, 4 priority factor $p_i (i = 1, 2, 3, 4)$ are considered for achievements of the aspired levels of the stated fuzzy goals. These are executed under four different priority structures, applying the software LINDO (Ver. 6.0). The results obtained for different priority structures are shown in Table 1. Now, observing the results in Table 2, Euclidean Distance Value 0.927 is found to be minimum. The results reflect that the priority structure under execution 2 & 4 are the appropriate one to obtain the most satisfactory solution. The optimal solution for the priority structures $\{P_1(d_2), P_2(d_1), P_3(d_3)\}$ is given by $x_{121} = 9.169975$; $x_{221} = 0.000032$; $x_{321} = 10.58$; $x_{331} = 20.25$; $x_{142} = 20.25$; $x_{213} = 9.58$; $x_{242} = 9.67$; $x_{323} = 10.67$.

\[ F_1(x) = 28942.93; \quad F_2(x) = 2457.48; \quad F_3(x) = 11563.79; \]

\[ \sum_{K=1}^{3} F^K(x) = 42964.2 \]

The membership functions are

$\mu_1(F_1(x))$, $\mu_2(F_2(x))$, $\mu_3(F_3(x)) = (0.073, 0.99, 1)$

<table>
<thead>
<tr>
<th>Execution Number</th>
<th>Priority Structure</th>
<th>Membership Value</th>
<th>Euclidean Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$[P_1(d_2), P_2(d_1), P_3(d_3)]$</td>
<td>(0.0019, 1, 0.99)</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>$[P_1(d_3), P_2(d_1), P_3(d_2)]$</td>
<td>(0.073, 0.99, 1)</td>
<td>0.927</td>
</tr>
<tr>
<td>3</td>
<td>$[P_1(d_2), P_2(d_3), P_3(d_1)]$</td>
<td>(0.0019, 1, 0.99)</td>
<td>0.998</td>
</tr>
<tr>
<td>4</td>
<td>$[P_1(d_3), P_2(d_2), P_3(d_1)]$</td>
<td>(0.073, 0.99, 1)</td>
<td>0.927</td>
</tr>
</tbody>
</table>

6. CONCLUSION

This paper proposed the solution procedure for MOSFTP using the priority based fuzzy goal programming approach with environmental cost and congestion time cost consideration along with the transportation cost. The numerical example shows that the consideration of environmental cost and congestion time cost are really changing the transportation policy. Also it helps the decision making unit to find out the appropriate priority structure. By making the users to pay for all the costs they cause, are a powerful instrument to change their travel decisions and reduce the environmental impacts of transport. The proposed method can be extended to other real world multi-objective...
programming problems involved with fuzzily described different parameters in the decision making context.

REFERENCES


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