A New ZCT and DHT Precoded OFDM System for M-QAM with Pulse Shaping: PAPR analysis

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Abstract

PAPR is one of the major drawbacks in the OFDM. In this paper we propose a study of PAPR and PAPR reduction technique using ZCT and DHT pre-coding methods. The ZCT is one type of technique used to reduce the PAPR analyzed. In ZCT we obtain the ZCT sequences from the ZCT kernel row wise filling which gives rise to the constant Envelope OFDM (CE-OFDM) system with the zero dB and ZCT sequences from column wise gives rise to the nearly 7.8 dB with the clip rate and the number of sub carriers used in the OFDM is \( N = 64 \) with QPSK or QAM modulation. And so in the present paper we analyze the various PAPR reduction techniques with the Root Raised Cosine (RRC) pulse shaping method. The final simulation results show the ZCT Row wise precoder based OFDM system has less PAPR than the ZCT Column wise precoder based OFDM, conventional OFDM.

Keywords -- Zadoff-Chu Matrix transforms (ZCT) system, Discrete Hartley Transform, conventional OFDM, PAPR (Peak to Average Power Ratio).

I. INTRODUCTION

Multicarrier transmission is also known as orthogonal Frequency Division multiplexing which is used for wireless and wire line digital communication systems because of its high speed data rates, high spectral efficiency, high quality service and frequency selective fading [9].

The interest in this technique is mainly due to the recent advantages in the Digital Signal Processing technology. International standards will use the OFDM for the high speed wireless communication or being established by the IEEE 802.20 [9]. OFDM is popular modulation technique for both wireless and wired communication.

It will provide the very high data rates over the harsh wireless channels that are characterized by the multi path fading. OFDM widely used in Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), Digital subscriber Lines (DSL), Wireless Local Area networks (WLANs), Wireless Metropolitan Area Networks (WMANs), and even it is used beyond the 3G Wide Area Networks (WANs).

OFDM provides Inter symbol Interference (ISI) by inserting the Guard Interval (GI) using the cyclic prefix (CP) and it will moderate the frequency selectivity of the Multi path channel with a simple equalizer [8]. One of the major drawbacks of Orthogonal Frequency Division Multiplexing is Peak to average Power Ratio (PAPR).

The High PAPR is occurred at the transmitter [10]. The high PAPR makes the OFDM sensitive to the non linear distortion caused by the transmitter power amplifier (PA). Without sufficient power back off the system suffers from the inter modulation distortion, performance degradation, etc [5, 10].

To reduce the PAPR we propose the different types of techniques like clipping, coding, peak windowing. Clipping is the simple technique which provides serious out of band radiation. Similarly the coding and peak windowing are the similar techniques which mainly used to reduce the PAPR but this type of techniques are not good for coding solutions. So for zero distortion of the OFDM signal the High Power amplifiers (HPA) need to be used for sufficient back off.

Even though large number of PAPR techniques proposed the pre-coding based technique is effectively used because of it provides the simple linear techniques. If we reducing the PAPR it means that we are reducing the OFDM system cost, reducing the complexity of the Analog to Digital (A/D), Digital to Analog (D/A) conversion, increasing the transmit power for the same range of receiving Signal to Noise Ratio (SNR).

In this paper we analyze the Zadoff-Chu Matrix Transform with Row wise precoder OFDM (ZCT-R-OFDM) system using the Root Raised cosine (RRC) Pulse shaping and the results are compared with the other pre-coding based OFDM systems [1, 2].

The below Paper is organized as follows section 2 discuss the OFDM block diagram and PAPR calculation, section 3 presents the proposed method (ZCT-R-OFDM), section 4 provides the project simulation results, and final section 5 concludes the project.
1. **OFDM Analysis and PAPR**

In OFDM initially the binary data is calculated and modulated using the Quadrature Amplitude Modulation or Quadrature Phase shift keying and it is converted to the parallel order which is mainly used to reduce complexity of the signal. After modulation the pilot tones are inserted in between the modulated data for securing the information, the modulated output is of complex vector of order N where N is sub carriers.

The parallel converted modulated data $X'(k)$ is passed to the Inverse Fourier Transform. The IFFT converts the modulated complex data to the time information $x(n)$ which is mainly used to reduce the complexity of the system and improves the fastness of the Algorithm [9].

The transformed data is passed to the receiver section through the channel. Before the passing through the channel Guard intervals are inserted in between the transformed data

$$x(n)$$ and forms $x1(n)$.

The Guard interval is used to reduce the overlapping in between the orthogonal signal. In the channel the Additive White Gaussian Noise (AWGN) is combined to the transformed data and it passed to the receiver section [5].

In the receiver section the guard interval is removed and demodulated the data after FFT, finally the output data is calculated

1.1 **OFDM Signal**

In OFDM signal we use N sub carriers for the message bit to transfer from transmitter to receiver. After QAM the message bits are of complex vector of size N

$$X(k) = [X1(k), X2(k), X3(k) ............]$$

Where X is passed through the IFFT block. The complex base band OFDM signal with N sub carriers is written as [10].

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k). e^{-j\frac{2\pi nk}{N}}$$

*Where $n = 0,1,2,3, .... N - 1$*

Here $j = \sqrt{-1}$ and $x_n$ is the IFFT block and $X(k)$ is the complex vector.

The PAPR is calculated as

$$PAPR = \frac{\max(x_n)^2}{E[x_n]^2}$$

Where $E[.]$ denotes the expectation of the signal.

The Complementary Cumulative Distribution Function (CCDF) for OFDM is calculated as

$$P(PAPR > PAPR0) = 1 - (1 - e^{-PAPR0})^N$$

Here $PAPR0$ is the clipping level and it exceeds some clip level $PAPR0$. 
2. System model

2.1 Zadoff-Chu (ZC) Sequences and Zadoff-Chu Matrix Transform (ZCT)

Zadoff–Chu (ZC) Sequences has an ideal periodic auto correlation with constant magnitude. The ZC Sequences of length N can be defined as [1]

\[
\alpha_n = \begin{cases} 
\frac{\left(\frac{k^2}{2} + qk\right)}{N} & \text{for } N \text{ is even} \\
\frac{e^{j\pi k}}{N} & \text{for } N \text{ is odd}
\end{cases}
\]

(4)

Where \( k = 0,1,2 \ldots N-1 \), \( q \) is the integer, \( r \) is the relatively prime to \( N \) and \( j = \sqrt{(-1)} \)

The ZCT kernel is obtained by reshaping the ZC sequences in the \( k = mL + l \)

\[
A = \begin{bmatrix}
a_0 & a_1 & a_2 & a_3 & \ldots & a_{(l-1)} \\
a_0 & a_1 & a_2 & a_3 & \ldots & a_{(l-1)} \\
a_0 & a_1 & a_2 & a_3 & \ldots & a_{(l-1)} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_0 & a_1 & a_2 & a_3 & \ldots & a_{(l-1)}
\end{bmatrix}
\]

The row variables are \( m \) and the column variables are \( l \), in other words \( N = L^2 \) point for long ZC sequences with row wise kernel matrix. The PAPR will reduce to zero dB in this case [1, 2].

If the kernel matrix is in column matrix then \( k = m + mL \) in this case PAPR will not reduce completely to zero dB but it reduces up to 7.8 dB.

2.2 ZCT Pre-coded OFDM system

The below shown Figure is the ZCT Pre-coded OFDM system. In this system the ZCT row wise Pre-coded matrix \( A \) is applied to the modulated constellation symbols before the Inverse Fast Fourier Transform (IFFT) to reduce the PAPR. In ZCT method the modulated data of complex vector of size \( L \) can be written as \( X = [X_0, X_1, \ldots, X_{L-1}]^T \).

After modulating the data ZCT pre-coding is applied to the modulated complex vector and transformed to the Inverse Fast Fourier Transform. The ZCT pre-coding can be written as [1, 2]

\[
Z = AX = [Z_0, Z_1, Z_2, \ldots, Z_{L-1}]^T
\]

Where \( Z \) is the ZCT precoder matrix with the size \( L \times L \) and \( X \) is the modulated complex data.

The ZCT precoder matrix can also be written as

\[
Z_m = \sum_{l=0}^{L-1} a_{m,l} \cdot X_l
\]

(5)

where \( m = 0,1,2,\ldots L-1 \)

\( a_{m,l} \) means \( m \)th row and \( l \)th column

From the above equation using the row wise shaping \( k = mL + l \) and putting the \( q = 0, r = 1 \) in the equation (3)

\[
Z_m = \sum_{l=0}^{L-1} \left( e^{j\frac{\pi mL}{L^2}} \right) X_l
\]

(6)

\[
\text{Fig 2: ZCT pre-coded OFDM system}
\]

\[
\text{Where } m = 0,1,2,\ldots L-1 \text{. The above equation provides the ZCT pre-coded constellation symbols.}
\]

The ZCT-Row Wise OFDM signal with the \( L \) sub carriers after IFFT is written as [1, 2]

\[
x_n = \frac{1}{\sqrt{L}} \sum_{m=0}^{L-1} Z_m \cdot e^{j\frac{2\pi mn}{L}},
\]

(7)

where \( n = 0,1,2,3L-1 \)

The pass band transmit signal \( x(t) \) of ZCT Row wise OFDM after Root Raised Cosine (RRC) pulse shaping and D/A of \( x_n \) can be written as [1]

\[
x(t) = e^{j\omega_c t} \sum_{n=0}^{L-1} x_n \cdot r(t - n\tau)
\]

(8)

Here \( \omega_c \) is the carrier frequency, \( r(t) \) is the base band pulse, \( \tau = \left( \frac{M}{N} \right) \). \( T \) is the compressed

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symbol duration in the seconds after inverse Fast Fourier Transform.

2.3 Root Raised Cosine

The Root Raised Cosine Pulse shaping filter can be defined as [2]

\[
r(t) = \frac{\sin\left(\frac{\pi t}{T}(1-\alpha)\right) + \alpha dt \cos\left(\frac{\pi t}{T}(1+\alpha)\right)}{\pi t(1-\frac{\pi t^2}{T^2})}
\]

(9)

\(0 \leq \alpha \leq 1\), where \(\alpha\) is the roll off factor.

The PAPR of ZCT-R-OFDM signal with pulse shaping can be written as

\[
PAPR = \left(\frac{\sum_{n=-\infty}^{\infty} |x(t)|^2 dt}{\sum_{n=-\infty}^{\infty} |x[n]|^2}\right)
\]

(10)

The PAPR of ZCT-R-OFDM without pulse shaping can be written as [1]

\[
PAPR = \left(\frac{\sum_{n=-\infty}^{\infty} |x[n]|^2}{\sum_{n=-\infty}^{\infty} |x[n]|^2}\right)
\]

(11)

After pre-coding using the ZCT the orthogonality is maintained between the symbols. Because the pre-coding matrix is auto orthogonal. The PAPR is better reduced in ZCT-R-OFDM with Pulse Shaping Algorithm than ZCT-R-OFDM without Pulse shaping.

2.4 Future scope

ZCT pre-coding matrix is used to reduce the PAPR using Row wise or Column wise matrix where the Row wise ZCT has better PAPR reduction compare to the column wise. Discrete Hartley Transform (DHT) is the pre-coding matrix used to reduce the PAPR efficiently than ZCT. DHT is the linear transform and the \(N\) real numbers are transformed to \(N\) real numbers where \(N\) is the number of sub carriers from the OFDM signal.

\[
\begin{align*}
H_k &= \sum_{n=-\infty}^{\infty} x_n \left[\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right)\right] \\
&= \sum_{n=-\infty}^{N-1} x(n) \cos\left(\frac{2\pi nk}{N}\right)
\end{align*}
\]

(12)

Where \(n\) is the integers from 0 to \(N-1\).

The pre-coding matrix \(p_{m,n}\) is placed in the given matrix and can be written as

\[
P = \begin{bmatrix}
p_{0,0} & p_{0,1} & \cdots & p_{0,N-1} \\
p_{1,0} & p_{1,1} & \cdots & p_{1,N-1} \\
p_{N-1,0} & p_{N-1,1} & \cdots & p_{N-1,N-1}
\end{bmatrix}
\]

(13)

The complex base band OFDM signal after IFFT can be written as the

\[
x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} P \ast X(k) e^{j \frac{2\pi nk}{N}}
\]

(14)

Where \(X\) is modulated complex data and \(P\) is the Pre-coding matrix.

The PAPR of OFDM signal is calculated as

\[
PAPR = \left(\frac{N+1}{N+2} \sum_{n=-\infty}^{\infty} |x[n]|^2\right)
\]

(15)

The advantage of using DHT pre-coding matrix is it has less complexity in calculating the pre-coding values and forming the matrix because in DHT the real numbers are used and transformed to the pre-coding matrix.

3. Simulation results

The complete analysis of the PAPR reduction is carried out in MATLAB software. It is used to find PAPR of ZCT-C-OFDM, ZCT-R-OFDM with the Root Raised Cosine Pulse Shaping. The PAPR is statically calculated using the Complementary Cumulative Distribution Function (CCDF). The CCDF of PAPR OFDM signal is used to find the probability of exceeding given \(PAPR_0\).

\[
Prob(PAPR > PAPR_0) = 1 - \left(1 - e^{-PAPR_0}\right)^N
\]

Fig: 3 CCDF comparison of ZCT-R-OFDM with ZCT-C-OFDM and conventional OFDM with Pulse shaping using \(N=64\) and \(\alpha = 0.2\).
In the Figure 3 the number of sub carriers \( N=64 \) and the roll off factor \( \alpha=0.2 \) are used for Pulse shaping and the ZCT Row wise OFDM has PAPR 4.5 dB and ZCT-C-OFDM has 7.5 dB and conventional OFDM has above 8 dB with the clip-rate \( 10^{-2} \).

![Graph for N=64 with alpha 0.2](image)

Fig: 4 CCDF comparison of ZCT-R-OFDM with ZCT-C-OFDM, DHT-OFDM and conventional OFDM with Pulse shaping using N=64 and \( \alpha=0.2 \).

In the Figure 4 the number of sub carriers \( N=64 \) and the roll off factor is \( \alpha=0.2 \) are used for Pulse shaping and the ZCT-R-OFDM system, ZCT-C-OFDM system, Conventional OFDM system PAPR has 5.3 dB, 9.5dB, 9.7dB with clip rate \( 10^{-2} \).

4. Conclusion

In this paper the PAPR analysis using ZCT-R-OFDM system with RRC pulse shaping is presented. The advantage of using the ZCT method is the PAPR reduction with less complexity. In the Figure 3 and Figure 4 it is analyzed that the ZCT-Row Wise-OFDM has lower PAPR than the ZCT-Column Wise-OFDM, conventional OFDM System. So it is concluded that the ZCT-R-OFDM has better PAPR reduction than the conventional OFDM, ZCT-C-OFDM and other types of PAPR reduction techniques. Another advantage of using the ZCT-R-OFDM is pulse shaping the symbol using RRC which does not require for increasing the power, complex optimization. Further to reduce PAPR than the ZCT we can use the Discrete Hartley Transform, which is less complex than the ZCT to calculate the Precoding matrix.

References


